**SECTION 4.3 - THE METHOD OF UNDETERMINED COEFFICIENTS**

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Here I solve the inhomogeneous fifth order ODE

$$y^v  \,\mbox{--}\,3y^{iv} + 3y''' \,\mbox{--}\, 3y'' + 2y' = x^5 + 6e^{7x}
\,\mbox{--}\, 2e^{\,\mbox{--}\,x}\cos(3x) + 9 \sin(7x).$$

**Solving by hand (letting MATLAB do each calculation)**

First I have MATLAB carry out the same steps I would do if I were solving it by hand. Then I see what happens if I ask MATLAB to solve the original equation.

Here is a MATLAB function (defined by a function-M file) which will apply the left side of the equation to a function f.

type diffop

function [L] = diffop(f,x)

% DIFFOP applies the differential operator L which takes y to y^v - 3y^{iv} + 3y'''- 3y'' + 2y' to the

% function f(x). It requires two arguments, a function and an independent variable.

L = diff(f,x,5) - 3\*diff(f,x,4) + 3\*diff(f,x,3) - 3\*diff(f,x,2) + 2\*diff(f,x);

The first step is to solve the corresponding homogeneous equation, by plugging

$$y = e^{rx}$$

into the left side.

syms r x

L = diffop(exp(r\*x),x)

L =

2\*r\*exp(r\*x) - 3\*r^2\*exp(r\*x) + 3\*r^3\*exp(r\*x) - 3\*r^4\*exp(r\*x) + r^5\*exp(r\*x)

I divide the left side by exp(rx). I have to give the **simplify** command to get MATLAB to carry out the division.

charpoly = simplify(L/exp(r\*x))

charpoly =

r\*(r^2 + 1)\*(r - 1)\*(r - 2)

Now I have to find the roots of the characteristic polynomial. **solve** assumes the right side of the equation is 0.

roots = solve(charpoly)

roots =

0

1

2

i

-i

I know that 1, exp(x), exp(2x), cos(x) and sin(x) form a fundamental set of solutions of the homogeneous equation.

The right side of the equation has 4 terms, each of a different form, so I break the inhomogeneous problem into 4 problems. For the first, the right side is x^5. So, since 1 solves the homogeneous equation, I look for a solution of the form y\_1 equal to x times a polynomial of degree 5. Here is a loop which builds it.

y\_1 = 0;

for k = 0:5

vars{k+1} = ['a' num2str(k)];

u = vars{k+1}\* x^(6-k);

y\_1 = u + y\_1;

end

y\_1

y\_1 =

a0\*x^6 + a1\*x^5 + a2\*x^4 + a3\*x^3 + a4\*x^2 + a5\*x

I plug it into the left side and subtract x^5. The expression is very lengthy. It helps to use **collect** to collect all the terms involving the same power of x (and factor the power out of the terms) and to use **pretty** so it doesn't run off the screen.

Y = collect(diffop(y\_1,x) - x^5,x); pretty(Y)

5 4 3

(12 a0 - 1) x + (10 a1 - 90 a0) x + (360 a0 - 60 a1 + 8 a2) x +

2

(180 a1 - 1080 a0 - 36 a2 + 6 a3) x + (720 a0 - 360 a1 + 72 a2 - 18 a3 + 4 a4) x +

120 a1 - 72 a2 + 18 a3 - 6 a4 + 2 a5

I want to solve the system of equations that I get from setting the coefficents equal to 0. I can either do this by copying and pasting the coefficients into the solve command or using a for loop to calculate the coefficients and set them equal to 0. Here I use a loop to do it. The value of the coefficient of x^j is the jth derivative of Y evaluated at 0. The corresponding equation is indexed by j+1.

for k = 1:6

eqn = subs(diff(Y,x,k-1),x,0);

eqns{k} = [char(eqn) '=0'];

end

Now I solve the resulting system.

range = 1:6;

sol = solve(eqns{range},vars{range});

for k = range

vals{k} = sol.(vars{k});

end

I substitute these into y\_1.

Y\_1 = subs(y\_1,vars,vals)

Y\_1 =

x^6/12 + (3\*x^5)/4 + (15\*x^4)/8 + (15\*x^3)/4 + (285\*x^2)/8 + (765\*x)/8

I verify that this is a solution. To get MATLAB to do the algebra after applying the left side to Y\_1, I have to use **simplify**.

simplify(diffop(Y\_1,x))

ans =

x^5

The right side of the second equation is 6 exp(7x), which doesn't solve the homogeneous equation so I look for a solution of the form

syms a

y\_2 = a\*exp(7\*x)

y\_2 =

a\*exp(7\*x)

I plug it into the left side of the equation and subtract 6 exp(7x). This time I want to collect the terms involving exp(7x).

Y = collect(diffop(y\_2,x)-6\*exp(7\*x),exp(7\*x))

Y =

(10500\*a - 6)\*exp(7\*x)

I need the coefficient of exp(7x).

C = coeffs(Y,exp(7\*x))

C =

10500\*a - 6

I solve this equation.

[a] = solve(C)

a =

1/1750

I evaluate y\_2 using this.

Y\_2 = eval(y\_2)

Y\_2 =

exp(7\*x)/1750

I verify that this is a solution.

diffop(Y\_2,x)

ans =

6\*exp(7\*x)

The third term on the right is - 2exp(-x)cos(3x) which doesn't solve the homoegeneous equation, so I look for a solution of the form

clear a

syms a b

y\_3 = a\*exp(-x)\*cos(3\*x)+b\*exp(-x)\*sin(3\*x)

y\_3 =

(a\*cos(3\*x))/exp(x) + (b\*sin(3\*x))/exp(x)

I plug this into the left side of the equation and subtract - 2exp(-x)cos(3x) When I do this, every term will contain a factor of exp(-x), so I divide the result by that.

Y = simplify((diffop(y\_3,x) + 2\*exp(-x)\*cos(3\*x))/exp(-x))

Y =

2\*cos(3\*x) + 330\*a\*sin(3\*x) - 300\*b\*sin(3\*x) - 300\*a\*cos(3\*x) - 330\*b\*cos(3\*x)

I could get the equations by evaluating Y and its derivative at 0. Instead I will use **coeffs** command. I notice that Y is a linear "polynomial" in cos(3x) and sin(3x), so I want the coefficients of cos(3x) and of sin(3x).

E1 = coeffs(Y,cos(3\*x))

E1 =

[ 330\*a\*sin(3\*x) - 300\*b\*sin(3\*x), 2 - 330\*b - 300\*a]

The coefficient of cos(3x) is the second term in E1

eq1 = E1(2)

eq1 =

2 - 330\*b - 300\*a

Now I find the coefficient of sin(3x).

E2 = coeffs(Y,sin(3\*x)); eq2 = E2(2)

eq2 =

330\*a - 300\*b

I solve the resulting equations using **solve**. The **solve** command assumes that the right side of each equation is 0.

[a,b] = solve(eq1,eq2); [a,b]

ans =

[ 2/663, 11/3315]

I evaluate y\_3 using this.

Y\_3 = eval(y\_3)

Y\_3 =

(2\*cos(3\*x))/(663\*exp(x)) + (11\*sin(3\*x))/(3315\*exp(x))

I verify that this solves the equation.

diffop(Y\_3,x)

ans =

-(2\*cos(3\*x))/exp(x)

The last term on the right is 9 sin(7x). This doesn't solve the homogeneous equation so I look for a solution of the form

clear a b

syms a b

y\_4 = a\*cos(7\*x) + b\*sin(7\*x)

y\_4 =

b\*sin(7\*x) + a\*cos(7\*x)

I plug this into the left side of the equation and subtract 9 sin(7x).

Y = diffop(y\_4,x) - 9\*sin(7\*x)

Y =

15792\*b\*cos(7\*x) - 15792\*a\*sin(7\*x) - 7056\*b\*sin(7\*x) - 7056\*a\*cos(7\*x) - 9\*sin(7\*x)

Again I use **coeffs** to get the equations.

E1 = coeffs(Y,cos(7\*x)); eq1 = E1(2)

eq1 =

15792\*b - 7056\*a

E2 = coeffs(Y,sin(7\*x)); eq2 = E2(2)

eq2 =

- 15792\*a - 7056\*b - 9

I solve the equations.

[a,b] = solve(eq1,eq2)

a =

-141/296800

b =

-9/42400

The resulting solution of the equation is

Y\_4 = eval(y\_4)

Y\_4 =

- (141\*cos(7\*x))/296800 - (9\*sin(7\*x))/42400

I verify that it solves the equation.

diffop(Y\_4,x)

ans =

9\*sin(7\*x)

The resulting particular solution of the original equation is obtained by adding these 4 solutions.

y\_p = Y\_1 + Y\_2 + Y\_3 + Y\_4; pretty(y\_p)

2 3 4

765 x 141 cos(7 x) exp(7 x) 9 sin(7 x) 2 cos(3 x) 285 x 15 x 15 x

----- - ------------ + -------- - ---------- + ---------- + ------ + ----- + ----- +

8 296800 1750 42400 663 exp(x) 8 4 8

5 6

3 x x 11 sin(3 x)

---- + -- + -----------

4 12 3315 exp(x)

**Solving with Matlab**

What happens if you let MATLAB solve the equation?

eqn = 'D5y - 3\*D4y + 3\*D3y - 3\*D2y + 2\*Dy= x^5 + 6\*exp(7\*x) - 2\*exp(-x)\*cos(3\*x) + 9\*sin(7\*x)';

Y = dsolve(eqn, 'x');

pretty(simple(Y))

765 x C18 141 cos(7 x) exp(7 x) 9 sin(7 x)

----- - --- - ------------ + -------- - ---------- + C19 cos(x) + C22 exp(x) +

8 2 296800 1750 42400

2 3 4 5 6

2 cos(3 x) 285 x 15 x 15 x 3 x x

C20 sin(x) + ---------- + ------ + ----- + ----- + ---- + -- + C21 exp(2 x) +

663 exp(x) 8 4 8 4 12

11 sin(3 x)

----------- + 765/16

3315 exp(x)

This is a general solution of the equation. I compare it with our particular solution.

simplify(Y - y\_p)

ans =

C19\*cos(x) - C18/2 + C22\*exp(x) + C20\*sin(x) + C21\*exp(2\*x) + 765/16

I get a solution of the homogeneous equation, which is what I should get.

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